LESSON

1.2 Sets of Real Numbers

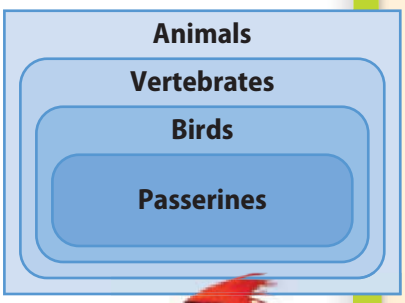
TEKS Number and operations—8.2.A Extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers.

ESSENTIAL QUESTION

How can you describe relationships between sets of real numbers?

Classifying Real Numbers

Biologists classify animals based on shared characteristics. A cardinal is an animal, a vertebrate, a bird, and a passerine.



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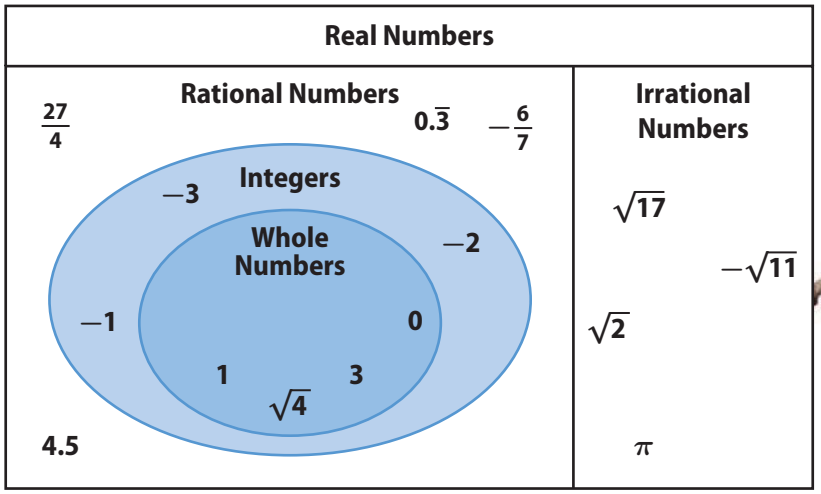
You already know that the set of rational numbers consists of whole numbers, integers, and fractions. The set of **real numbers** consists of the set of rational numbers and the set of irrational numbers.



Passerines, such as the cardinal, are also called "perching birds."

EXAMPLE 1

TEKS 8.2.A

Write all names that apply to each number.

A $\sqrt{5}$

5 is a whole number that is not a perfect square.

irrational, real

B −17.84

−17.84 is a terminating decimal.

rational, real

C $\frac{\sqrt{81}}{9}$

$\frac{\sqrt{81}}{9} = \frac{9}{9} = 1$

whole, integer, rational, real

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Math Talk
Mathematical Processes
What types of numbers are between 3.1 and 3.9 on a number line?

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YOUR TURN

Write all names that apply to each number.

1. A baseball pitcher has pitched $12\frac{2}{3}$ innings.

_____

2. The length of the side of a square that has an area of 10 square yards. _____

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Understanding Sets and Subsets of Real Numbers

By understanding which sets are subsets of types of numbers, you can verify whether statements about the relationships between sets are true or false.

EXAMPLE 2

TEKS 8.2.A

Tell whether the given statement is true or false. Explain your choice.

A All irrational numbers are real numbers.

True. Every irrational number is included in the set of real numbers. Irrational numbers are a subset of real numbers.

B No rational numbers are whole numbers.

False. A whole number can be written as a fraction with a denominator of 1, so every whole number is included in the set of rational numbers. Whole numbers are a subset of rational numbers.

Math Talk
Mathematical Processes

Give an example of a rational number that is a whole number. Show that the number is both whole and rational.

YOUR TURN

Tell whether the given statement is true or false. Explain your choice.

3. All rational numbers are integers.

_____

_____

_____

4. Some irrational numbers are integers.

_____

_____

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Identifying Sets for Real-World Situations

Real numbers can be used to represent real-world quantities. Highways have posted speed limit signs that are represented by natural numbers such as 55 mph. Integers appear on thermometers. Rational numbers are used in many daily activities, including cooking. For example, ingredients in a recipe are often given in fractional amounts such as $\frac{2}{3}$ cup flour.

EXAMPLE 3

Real World

TEKS 8.2.A

Identify the set of numbers that best describes each situation. Explain your choice.

A the number of people wearing glasses in a room

The set of whole numbers best describes the situation. The number of people wearing glasses may be 0 or a counting number.

B the circumference of a flying disk has a diameter of 8, 9, 10, 11, or 14 inches

The set of irrational numbers best describes the situation. Each circumference would be a product of π and the diameter, and any multiple of π is irrational.

My Notes

YOUR TURN

Identify the set of numbers that best describes the situation. Explain your choice.

5. the amount of water in a glass as it evaporates

_____

_____

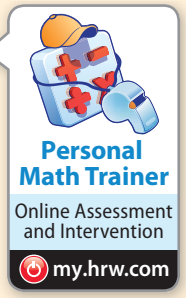
_____

6. the number of seconds remaining when a song is playing, displayed as a negative number

_____

_____

_____



LESSON 1.3

Ordering Real Numbers

TEKS **Number and operations—8.2.D** Order a set of real numbers arising from mathematical and real-world contexts. *Also 8.2.B*

ESSENTIAL QUESTION

How do you order a set of real numbers?

Comparing Irrational Numbers

Between any two real numbers is another real number. To compare and order real numbers, you can approximate irrational numbers as decimals.

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EXAMPLE 1

TEKS 8.2.B

Compare $\sqrt{3} + 5 \bigcirc 3 + \sqrt{5}$. Write $<$, $>$, or $=$.

STEP 1 First approximate $\sqrt{3}$.

$\sqrt{3}$ is between 1 and 2, so $\sqrt{3} \approx 1.5$.

Next approximate $\sqrt{5}$.

$\sqrt{5}$ is between 2 and 3, so $\sqrt{5} \approx 2.5$.

Use perfect squares to estimate square roots.
$1^2 = 1 \quad 2^2 = 4 \quad 3^2 = 9$

STEP 2 Then use your approximations to simplify the expressions.

$\sqrt{3} + 5$ is between 6 and 7

$3 + \sqrt{5}$ is between 5 and 6

So, $\sqrt{3} + 5 > 3 + \sqrt{5}$

My Notes

Reflect

1. If $7 + \sqrt{5}$ is equal to $\sqrt{5}$ plus a number, what do you know about the number? Why?

_____

2. What are the closest two integers that $\sqrt{300}$ is between?

_____

YOUR TURN

Compare. Write $<$, $>$, or $=$.

3. $\sqrt{2} + 4 \bigcirc 2 + \sqrt{4}$

4. $\sqrt{12} + 6 \bigcirc 12 + \sqrt{6}$

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Ordering Real Numbers

You can compare and order real numbers and list them from least to greatest.

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My Notes

EXAMPLE 2

TEKS 8.2.D

Order $\sqrt{22}$, $\pi + 1$, and $4\frac{1}{2}$ from least to greatest.

STEP 1 First approximate $\sqrt{22}$.

$\sqrt{22}$ is between 4 and 5. Since you don't know where it falls between 4 and 5, you need to find a better estimate for $\sqrt{22}$ so you can compare it to $4\frac{1}{2}$.

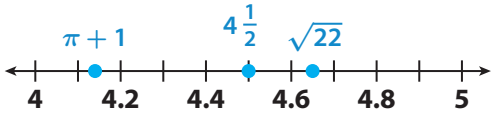
To find a better estimate of $\sqrt{22}$, check the squares of numbers close to 4.5.

$4.4^2 = 19.36$ $4.5^2 = 20.25$ $4.6^2 = 21.16$ $4.7^2 = 22.09$

$\sqrt{22}$ is between 4.6 and 4.7, so $\sqrt{22} \approx 4.65$.

An approximate value of π is 3.14. So an approximate value of $\pi + 1$ is 4.14.

STEP 2 Plot $\sqrt{22}$, $\pi + 1$, and $4\frac{1}{2}$ on a number line.



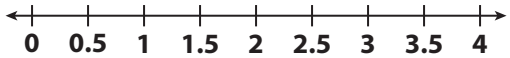
Read the numbers from left to right to place them in order from least to greatest.

From least to greatest, the numbers are $\pi + 1$, $4\frac{1}{2}$, and $\sqrt{22}$.

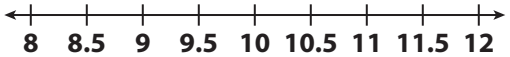
YOUR TURN

Order the numbers from least to greatest. Then graph them on the number line.

5. $\sqrt{5}$, 2.5, $\sqrt{3}$ _____



6. π^2, 10, $\sqrt{75}$ _____



Math Talk
Mathematical Processes

If real numbers *a*, *b*, and *c* are in order from least to greatest, what is the order of their opposites from least to greatest? Explain.

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Ordering Real Numbers in a Real-World Context

Calculations and estimations in the real world may differ. It can be important to know not only which are the most accurate but which give the greatest or least values, depending upon the context.

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EXAMPLE 3

Real World

TEKS 8.2.D

Four people have found the distance in kilometers across a canyon using different methods. Their results are given in the table. Order the distances from greatest to least.

Distance Across Quarry Canyon (km)			
Juana	**Lee Ann**	**Ryne**	**Jackson**
$\sqrt{28}$	$\frac{23}{4}$	$5.\overline{5}$	$5\frac{1}{2}$

STEP 1 Approximate $\sqrt{28}$.

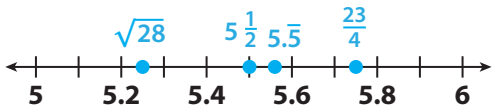
$\sqrt{28}$ is between 5.2 and 5.3, so $\sqrt{28} \approx 5.25$.

$\frac{23}{4} = 5.75$

$5.\overline{5}$ is 5.555…, so $5.\overline{5}$ to the nearest hundredth is 5.56.

$5\frac{1}{2} = 5.5$

STEP 2 Plot $\sqrt{28}$, $\frac{23}{4}$, $5.\overline{5}$, and $5\frac{1}{2}$ on a number line.



From greatest to least, the distances are:

$\frac{23}{4}$ km, $5.\overline{5}$ km, $5\frac{1}{2}$ km, $\sqrt{28}$ km.

YOUR TURN

7. Four people have found the distance in miles across a crater using different methods. Their results are given below.

Jonathan: $\frac{10}{3}$, Elaine: $3.\overline{45}$, José: $3\frac{1}{2}$, Lashonda: $\sqrt{10}$

Order the distances from greatest to least.

_____

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