LESSON **1.1**

Rational and Irrational Numbers

TEKS Number and operations—8.2.B Approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line.

ESSENTIAL QUESTION

How do you express a rational number as a decimal and approximate the value of an irrational number?

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Expressing Rational Numbers as Decimals

A **rational number** is any number that can be written as a ratio in the form $\frac{a}{b}$, where a and b are integers and b is not 0. Examples of rational numbers are 6 and 0.5.

6 can be written as $\frac{6}{1}$ 0.5 can be written as $\frac{1}{2}$

Every rational number can be written as a terminating decimal or a repeating decimal. A **terminating decimal**, such as 0.5, has a finite number of digits. A **repeating decimal** has a block of one or more digits that repeat indefinitely.

EXAMPLE 1

TEKS Prep for 8.2.B

Write each fraction as a decimal.

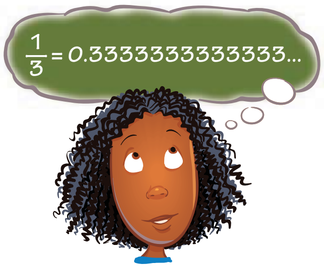
A $\frac{1}{4}$

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Remember that the fraction bar means "divided by." Divide the numerator by the denominator.

Divide until the remainder is zero, adding zeros after the decimal point in the dividend as needed.

$\frac{1}{4} = 0.25$



B $\frac{1}{3}$

$$\begin{array}{r} 0.333 \\ 3\overline{)1.000} \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

Divide until the remainder is zero or until the digits in the quotient begin to repeat.

Add zeros after the decimal point in the dividend as needed.

When a decimal has one or more digits that repeat indefinitely, write the decimal with a bar over the repeating digit(s).

$\frac{1}{3} = 0.\overline{3}$

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YOUR TURN

Write each fraction as a decimal.

1. $\frac{5}{11}$ ______ **2.** $\frac{1}{8}$ ______ **3.** $2\frac{1}{3}$ ______

Finding Square Roots of Perfect Squares

A number that is multiplied by itself to form a product is a **square root** of that product. Taking the square root of a number is the inverse of squaring the number.

$6^2 = 36$ 6 is one of the square roots of 36

Every positive number has two square roots, one positive and one negative. The radical symbol $\sqrt{\ }$ indicates the nonnegative or **principal square root** of a number. A minus sign is used to show the negative square root of a number.

$\sqrt{36} = 6$ $-\sqrt{36} = -6$

The number 36 is an example of a perfect square. A **perfect square** has integers as its square roots.

EXAMPLE 2

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Find the two square roots of each number.

A 169

$\sqrt{169} = 13$ 13 is a square root, since $13 \cdot 13 = 169$.

$-\sqrt{169} = -13$ -13 is a square root, since $(-13)(-13) = 169$.

B $\frac{1}{25}$

Since 1 and 25 are both perfect squares, you can find the square root of the numerator and the denominator.

$\sqrt{\frac{1}{25}} = \frac{1}{5}$ 1 is a square root of 1, since $1 \cdot 1 = 1$, and 5 is a square root of 25, since $5 \cdot 5 = 25$.

$-\sqrt{\frac{1}{25}} = -\frac{1}{5}$ $-\frac{1}{5}$ is a square root, since $\left(-\frac{1}{5}\right) \cdot \left(-\frac{1}{5}\right) = \frac{1}{25}$.

Math Talk
Mathematical Processes
Can you square an integer and get a negative number? Explain.

Reflect

4. **Analyze Relationships** How are the two square roots of a positive number related? Which is the principal square root?

______

5. Is the principal square root of 2 a whole number? What types of numbers have whole number square roots?

______

YOUR TURN

Find the two square roots of each number.

6. 64 ________ **7.** 100 ________ **8.** $\frac{1}{9}$ ________

9. A square garden has an area of 144 square feet. How long is each side?

____

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EXPLORE ACTIVITY 1

TEKS 8.2.B

Estimating Irrational Numbers

Irrational numbers are numbers that are not rational. In other words, they cannot be written in the form $\frac{a}{b}$, where a and b are integers and b is not 0.

Estimate the value of $\sqrt{2}$.

A Since 2 is not a perfect square, $\sqrt{2}$ is irrational.

B To estimate $\sqrt{2}$, first find two consecutive perfect squares that 2 is between. Complete the inequality by writing these perfect squares in the boxes.

$\square < 2 < \square$

C Now take the square root of each number.

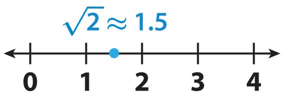
$\sqrt{\square} < \sqrt{2} < \sqrt{\square}$

D Simplify the square roots of perfect squares.

$\square < \sqrt{2} < \square$

$\sqrt{2}$ is between ______ and ______.

E Estimate that $\sqrt{2} \approx 1.5$.



F To find a better estimate, first choose some numbers between 1 and 2 and square them. For example, choose 1.3, 1.4, and 1.5.

$1.3^2 =$ ______ $1.4^2 =$ ______ $1.5^2 =$ ______

Is $\sqrt{2}$ between 1.3 and 1.4? How do you know?

____

Is $\sqrt{2}$ between 1.4 and 1.5? How do you know?

____

$\sqrt{2}$ is between ______ and ______, so $\sqrt{2} \approx$ ______.

G Locate and label this value on the number line.

Reflect

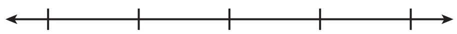
10. How could you find an even better estimate of $\sqrt{2}$?

____

11. Find a better estimate of $\sqrt{2}$. Draw a number line and locate and label your estimate.

$\sqrt{2}$ is between ______ and ______, so $\sqrt{2} \approx$ ______.

12. Estimate the value of $\sqrt{7}$ to the nearest 0.05. Draw a number line and locate and label your estimate.



$\sqrt{7}$ is between ______ and ______, so $\sqrt{7} \approx$ ______.

EXPLORE ACTIVITY 2 Real World

TEKS 8.2.B

Approximating π

The number π, the ratio of the circumference of a circle to its diameter, is an irrational number. It cannot be written as the ratio of two integers.

In this activity, you will explore the relationship between the diameter and circumference of a circle.

A Use a tape measure to measure the circumference and the diameter of four circular objects using metric measurements. To measure the circumference, wrap the tape measure tightly around the object and determine the mark where the tape starts to overlap the beginning of the tape. When measuring the diameter, be sure to measure the distance across the object at its widest point.

B Record the circumference and diameter of each object in the table.

Object	Circumference	Diameter	$\frac{\text{circumference}}{\text{diameter}}$

C Divide the circumference by the diameter for each object. Round each answer to the nearest hundredth and record it in the table.

D Describe what you notice about the ratio of circumference to diameter.

Reflect

13. What does the fact that π is irrational indicate about its decimal equivalent?

14. Plot π on the number line.

15. **Explain Why...** A CD and a DVD have the same diameter. Explain why they have the same circumference.

Guided Practice

1. **Vocabulary** Square roots of numbers that are not perfect squares are

____

Write each fraction as a decimal. (Example 1)

2. $\frac{7}{8}$ __________ **3.** $\frac{17}{20}$ __________ **4.** $\frac{18}{25}$ __________

5. $2\frac{3}{8}$ __________ **6.** $5\frac{2}{3}$ __________ **7.** $2\frac{4}{5}$ __________

Find the two square roots of each number. (Example 2)

8. 49 __________ **9.** 144 __________ **10.** 400 __________

11. $\frac{1}{16}$ __________ **12.** $\frac{4}{9}$ __________ **13.** $\frac{9}{4}$ __________

Approximate each irrational number to the nearest 0.05 without using a calculator. (Explore Activity 1)

14. $\sqrt{34}$ __________ **15.** $\sqrt{82}$ __________ **16.** $\sqrt{45}$ __________

17. $\sqrt{104}$ __________ **18.** $-\sqrt{71}$ __________ **19.** $-\sqrt{19}$ __________

20. **Measurement** Complete the table for the measurements to estimate the value of π. Round to the nearest tenth. (Explore Activity 2)

Circumference (in.)	Diameter (in.)	$\frac{\text{circumference}}{\text{diameter}}$
70	22	
110	35	
130	41	
200	62	

Describe what you notice about the ratio of circumference to diameter.

____

ESSENTIAL QUESTION CHECK-IN

21. Describe how to approximate the value of an irrational number.

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